



Development of an improved dynamic model of a Stirling engine and a performance analysis of a cogeneration plant



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H I G H L I G H T S

- Optimisation of a lump transient model based on a SE cogeneration unit.
- Performance analysis of a SE cogeneration unit.
- Variation of the energy to thermal ratio (ETTR).
- Calibration of a dynamic model using TRNSYS and GenOpt.

A R T I C L E I N F O

Article history:

Received 7 May 2014

Accepted 30 July 2014

Available online 13 August 2014

Keywords:

Stirling engine

Micro-cogeneration

TRNSYS

Optimisation

GenOpt

Energy to thermal ratio (ETTR)

A B S T R A C T

In this paper, the authors develop a dynamic model of a commercial micro-combined heat and power (mCHP) unit and analyse its dynamic behaviour when the engine is running at different mass flow inputs. The simulation predicts with a low root mean bias error (RMSE) the most important outputs from the cogeneration unit during the starting, steady-state and stopping periods. Furthermore, the presented transient model reproduces the behaviour of the cogeneration unit when the fuel and air mass flows are changing. The obtained results are discussed, and the different possibilities for the variation of the thermal to power ratio are analysed. These combinations include the variation of the flow distribution inside the machine and the position of the exhaust heat exchanger. The power to thermal ratio can be modified between 0.15 and 0.26 for these combinations. The performance of the engine and the variation of the heat source temperature are also analysed theoretically. The simulation results conclude that an important saving could be obtained when the electrical to thermal ratio (ETTR) is tracked for the power or thermal demands from a dwelling.

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1. Introduction

Stirling engines (SEs) will represent an important micro-cogeneration device in the next few years because classical energy resources are progressively increasing in cost. The external combustion of SEs allows the utilisation of different heat sources, which makes SEs more independent of energy market fluctuations.

Moreover the utilisation of mCHP units is expanding in the domestic sector because different technologies are being developed

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[1], and the emerging heat flow market represents an important role in district heating systems. Among others, the Stirling engine (SE), internal combustion engine (ICE), organic Rankine cycle (ORC) and fuel cell (FC) technologies have great potential for development [1]. The electric power output of these domestic devices is normally in the range of 0–2 kW_e, although the electrical to thermal ratio (ETTR) varies between 0.08 and 0.59 depending on the technology. In SEs, this variation is still smaller. Normally, this ratio is reduced to the interval 0–0.3 in the mCHP unit with Stirling technology. Although cogeneration capabilities are the most common, some ICE devices can also supply the dwelling with cooling during the summer time using tri-generation capabilities [2,3].

There are different studies that simulate the operation of a SE cogeneration unit. Some studies may discretise the performance time into different running modes [4,5] to take into account the

transient behaviour. Other studies may integrate the cogeneration unit into a building simulation environment to analyse the coupling with a dwelling during a meteorological year [6]. Other works may focus on this coupling and may analyse different SE devices of the building sector for the seasonal performance and the desired characteristics from the cogeneration unit [7,8].

The upper limit of the Stirling engine efficiency is bounded by Carnot [9] and cannot be exceeded. Depending on the characteristics from the working fluid (principally conductivity, specific heat, viscosity and density) within the closed thermodynamic cycle and the working pressures and temperatures, the electrical to thermal ratio can be strongly modified, and power efficiencies between 10 and 60% could be obtained [10]. Normally, this ratio is constructively designed and depends principally on swept and dead volumes, on regenerator characteristics and on the selected working fluid [6]. However, these variables are difficult to change during the running mode. Some SEs can externally modify the stroke, the phase angle or the dead volume from the piston, but these capacities require an extra mechanical complexity, which for many instances is not justified. Moreover, there are some types of SEs where these modifications are impossible. Although the phase angle variation is one of the best methods for power control, the double-acting engine in this study does not allow this possibility [11].

The power control of the SEs can also be performed by varying the operational characteristics, such as pressure, temperature, speed or electric load. Occasionally, the type of fuel used or the engine configuration restricts the power control method [1]. Other authors analyse the behaviour of the SEs through the variation in the characteristics of the working fluid. This working fluid is simulated as a lumped-mass transient model. The shaft power output is investigated when different parameters from the SE are changed [12]. This model reveals that as the operating speed increases, the difference in temperature between the source and the sink decreases. Consequently, the shaft power output also declines, but a maximum is reached at a critical speed.

A lump transient model is also a valid method that is often used to simulate the cold start modelling and transient response of thermal engines [13].

Stirling engines were also dynamically simulated using dynamic models based on experimental data [14] or by determining the parameters that the IEA/ECBS Annex 42 specifies [15], although these methods are sometimes cumbersome and difficult to undergo.

Recent studies analyse the feeding of a family house by means of external and internal combustion engines (ICEs), and the results are compared with traditional power production systems [16]. The three selected micro-cogeneration devices were operated in heat manage mode to control the temperature of a thermal storage tank, and only the ETTR that was established by the manufacturers was used. The modification of this ratio becomes especially interesting in isolated applications, where a net connection or district heating is not available or when the excess energy is under-paid.

Comparing ICEs with external combustion engines, which are the most common technologies in mCHP units, the ICEs present higher mechanical losses because of more moving parts and higher noise and vibrations due to the explosions and high operating speeds. ICEs were also calibrated using the IEA Annex 42 [17] but as previously mentioned, this method is time-consuming and requires the estimation of more than one hundred coefficients, which are derived from experimental data.

Most of the SE cogeneration units operate at full capacity and try to work at the most efficient working point. While the temperature from the heat source is kept as high as possible, the mean cycle pressure is also kept high and consequently the power output

because they are directly proportional. Normally, there is a maximum power at a determined engine speed. This maximum will depend on the working pressure, source temperature, the working fluid or the regenerator wire mesh [18]. This work reveals that a lineal relation also exists between the engine speed and the obtained torque in the shaft.

Thermal losses can have an enormous influence on the results. Although some analyses do not consider these losses, most of them include irreversibilities and thermal losses into the cooler, the heater and the regenerator [19,20]. Numerous works analyse the behaviour of different SE configurations based on different thermodynamic assumptions [21,22]. Others analyses try to validate the theoretical models, which simulate the thermodynamic behaviour of an experimental SE unit [18,23].

Although numerous works have centred around the Whispergen unit [5,10], no analysis has been oriented to create easy use and to calibrate the model's integration into a simulation tool environment with a transient and a variation of the ETTR capacities.

Therefore, the present paper describes and simulates a commercial SE design with a double-acting alpha configuration. A previous lump model from the authors [24] was improved because when this model was applied in transient test varying the mass airflow going into the combustion chamber and consequently the fuel consumption and source temperature, it was discovered that this model did not perfectly follow the transient behaviour from the machine. Furthermore, the analysis that was undergone in this article includes a transient study of the cogeneration device and different possibilities of varying the ETTR during the running mode. Consequently, this ratio could be slightly modified attending to household demand or user's interests. That modulation does not include variations in the internal parts of the engine such as the material or porosity of the regenerator or modifications in SE's geometry. Cycle mathematical analyses were not considered and only variations in mass flows or source temperatures including different heat exchanger arrangement have been evaluated.

Furthermore, this transient model could be applied in different scenarios especially in isolated applications such as the presented in Refs. [25] or [26]. The model is also interesting for grid connected or heat flow market applications where the variation of the ETTR could be applied depending on market prices or user's demands at any time.

In the first sections of this article, the main characteristics of the selected Stirling engine are explained focussing on the power output at different source temperatures. Subsequently, the improved transient model based on two lumps together with the procedure to integrate it in the simulation environment TRNSYS are explained. An analysis about the main exchanger energy inside the cogeneration system is made then and the new proposed model is presented. Finally, the experimental and simulated results are discussed and the energy delivered during the shutdown process is analysed.

2. Materials and methodology

Firstly, different tests were carried out for a test bench principally consisting of a commercial Stirling device from the manufacturer Whispergen together with an electric circuit and a thermal circuit to simulate the hypothetical electric and thermal loads from a dwelling.

These tests were performed in transient mode and were running on diesel and with different air mass flow inputs. The procedure was similar for all of the one-hour length tests. These data were used to calibrate the lumped mass model developed in the previous article by the authors in Ref. [24] and was modified to better fit the transient behaviour. In this way, the new dynamic

system presents a better transient response when the airflow and consequently, the fuel input entering into the burner is varied.

Once the model is validated, the cogeneration unit is analysed, and the possibilities for regulating the electrical to thermal ratio (ETTR) are investigated. The easiest way to achieve this regulation using the same working fluid and Stirling morphology is to adjust the hot and cold source temperatures, which directly modifies the maximum efficiency from Carnot's cycle [9]. The sink temperature is delimited by the type of heating system adopted, but the hot source temperature could be slightly changed for the loads or the consumer's interests.

3. Problem definition

The need to research cogeneration devices efficiency in its final location caused the emergence of transient models. Normally, these transient models require the calculation of numerous constants which involves time dedication. The present work proposes an easy-to-calibrate model which is perfectly integrated in a simulation environment (i.e. TRNSYS software) for subsequent simulation. Moreover the possibilities of variation of the ETTR from the cogeneration unit are investigated. Thus, one proposed scenario analyses position variation of the exhaust gases heat recovery exchanger. This modification incorporates important changes in the ETTR which may be used to increase the efficient of the system in particular cases.

Constant ETTR is normally used in research because units are optimised to run within the maximum efficiency. However the cogeneration unit under study was developed to run in an autonomous way; so the ETTR variation capacity becomes in a very interesting feature.

4. Characteristics of the SE

The selected SE is a commercial cogeneration device from the manufacturer Whisper Tech Limited. The model is the PPS16-24MD that corresponds with a double-acting alpha-type model, which uses a wobble yoke mechanism to transform the reciprocating movement of the pistons in a rotational one to move an alternator. This model burns diesel, but it can be easily transformed to run with a different fuel. Some projects made this engine work with bio-oil or ethanol [10], and the manufacturer actually commercialised a similar device that was fed by natural gas.

The engine presents four cylinders with pressurised nitrogen at a maximum of 28 bars. Table 1 shows the principal SE parameters taken from Ref. [27]. The dead volumes are approximately an order of magnitude less than the swept volumes and although the presence of dead volumes reduces the power output from the system, the efficiencies are not always reduced [11].

Fig. 1 represents the distribution of the different volumes and the heat exchangers inside the SE. The wobble yoke mechanism together with the alternator is also represented. The double-acting alpha-type model is a special arrangement where the lower part of each cylinder acts as the displacer of the adjacent piston as shown in Fig. 1.

The experimental installation of the test bench, the data acquisition system together with sensors' distribution and time step selection were explained in a previous article from the authors of [26].

5. Analysis and simulation of the power output

The Schmidt analysis is a common theoretical method where the different configurations of SEs were highly analysed. It was performance the Schmidt analysis into the Whispergen unit to

Table 1
Variables and Stirling engine parameters.

Description	Variables	Value	Units
Specific heat of flue gases	C_{p_gas}	1.107	[kJ/kg K]
Specific heat of water	C_{p_water}	4.18	[kJ/kg K]
Thermal capacitance of block	MC_{block}	13.625	[kJ/K]
Thermal capacitance of burner	MC_{burner}	54.75	[kJ/K]
Air mass flow rate	$\dot{m}_{air}$		[kg/s]
Diesel mass flow rate	$\dot{m}_{fuel}$		[kg/s]
Flue gases mass flow rate	$\dot{m}_{gas}$		[kg/s]
Heating system water mass flow rate	$\dot{m}_{water}$		[kg/s]
Lower heating value	LHV	42,800	[kJ/kg]
Energy produced with gas oil combustion	Q_{comb}		[kJ]
Energy absorbed by the water from the lumped mass	Q_{water}		[kJ]
Root mean square error	RMSE		[–]
Ambient temperature	T_{amb}		[K]
Experimental temperature	T_{exp}		[K]
Exhaust gases temperature	T_{exh}		[K]
Flue gases temperature	T_{gas}		[K]
Temperature of lumped block mass	T_{block}		[K]
Temperature of lumped burner mass	T_{burner}		[K]
Water temperature at heat exchanger outlet	T_{out}		[K]
Water temperature returning from heating system	T_{ret}		[K]
Simulated temperature	T_{sim}		[K]
Overall heat transfer coefficient from burner to water block	$(U \cdot A)_{lumps}$		[W/K]
Overall heat transfer coefficient for gases-water heat exchanger	$(U \cdot A)_{HE}$		[W/K]
Density of lumped mass	ρ		[kg/m ³]
Bore	B	4.200E-02	[m]
Stroke	S	2.000E-02	[m]
Piston area	A_P	1.385E-03	[m ²]
Swept volume expansion piston	V_{SE}	3.414E-05	[m ³]
Swept volume compression piston	V_{SC}	2.813E-05	[m ³]
Dead volume of expansion space	V_{DE}	3.592E-06	[m ³]
Dead volume of compression space	V_{DC}	6.751E-07	[m ³]
Heater volume	V_H	9.180E-06	[m ³]
Cooler volume	V_K	8.269E-06	[m ³]
Regenerator volume	V_R	1.745E-05	[m ³]
Total mass of working gas	M	3.750E-04	[kg]
Thermal resistance from the hot side	R_H		[m ² K/W]
Heat flow going into the expansion space	q_{in}		[W/m ²]
Convection coefficient in the expansion space	h_H		[W/m ² K]
Thermal resistance from the cold side	R_C		[m ² K/W]
Heat flow going out the compression space	q_{out}		[W/m ²]
Convection coefficient in the compression space	h_C		[W/m ² K]
Expansion space gas temperature	T_H	9.750E+02	[K]
Compression space gas temperature	T_C	3.600E+02	[K]
Regenerator space gas temperature	T_R	6.675E+02	[K]
Phase angle	A	9.000E+01	[°]
Temperature ratio	T	3.692E-01	[–]
Swept volume ratio	V	8.239E-01	[–]
Dead volume ratio	X	1.880E-01	[–]
Engine speed	N	1.500E+03	[rpm]
Engine speed	N	2.500E+01	[Hz]
Indicated expansion energy	W_E	3.651E+01	[J]
Indicated compression energy	W_C	1.348E+01	[J]
Indicated energy	W_i	2.303E+01	[J]
Number of cylinders	Z	4.000E+00	[–]
Specific heat at constant pressure [Nitrogen 300 K]	C_{pn}	1.039E+03	[J/kg K]
Specific heat at constant volume [Nitrogen 300 K]	C_{vn}	7.430E+02	[J/kg K]
Gas constant	R	2.968E+02	[J/kg K]
Gas constant	R	8.314E+00	[J/K mol]
V_{DE}/V_{SE}	X_{DE}	1.052E-01	[–]

Table 1 (continued)

Description	Variables	Value	Units
V_{DC}/V_{SE}	X_{DC}	1.977E-02	[-]
V_R/V_{SE}	X_R	5.110E-01	[-]
Molar mass N_2	M_{N_2}	2.801E-02	[kg/mol]
Thermal conductivity N_2	λ	2.598E-02	[W/K m]
Number of poles	P	12	[-]

better understand its behaviour when expansion temperature is varied.

This mathematical model does not take into account the irreversibility introduced by friction, throttling or pressure losses because it is a simple method where the working gas is considered an ideal gas. Although there are different improvements to this theory as introduced by Gustav, Finkelstein, Senft [1] or Petresku [28] that attempt to include the internal losses generated into the engine, our dynamic lump model tries to simulate the global behaviour of the SE in the simplest way. Although the temperatures of the heater, cooler and regenerator vary with time because of their transient behaviour, they are assumed uniform along the different chambers. Only the expansion and compression space volume will vary with time and angle. The heater, cooler and regenerator volumes together with the swept and dead volumes will be considered as constants.

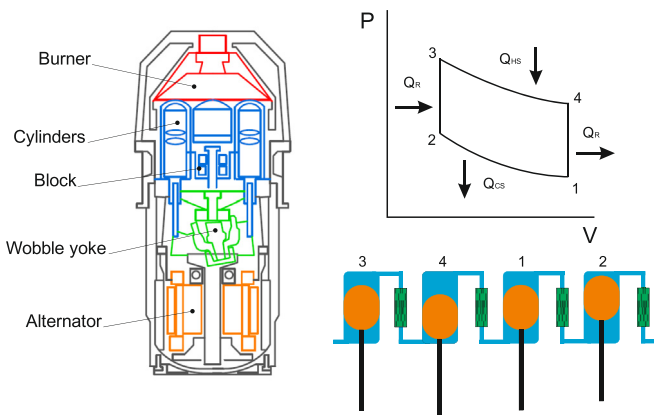
Although the Schmidt analysis does not include the internal hydraulic or thermal losses among others irreversibilities, it will be precise enough to explain the transient Whispergen performance during the variation of the source temperature. The principal equations utilised in the Schmidt analysis are the expressions from (1) to (6).

$$V_E = \frac{V_{SE}}{2} (1 - \cos \varnothing) + V_{DE} + V_H \quad (1)$$

$$V_C = \frac{V_{SC}}{2} (1 - \cos (\varnothing - \alpha)) + V_{DC} + V_K \quad (2)$$

$$a = \tan^{-1} \frac{v \cdot \sin \alpha}{t + \cos \alpha} \quad (3)$$

$$S = t + 2tX_{DE} + \frac{4tX_R}{1+t} + v + 2X_{DC} \quad (4)$$

**Fig. 1.** Stirling engine distribution and of four-cycle double acting alpha configuration.

$$B = \sqrt{t^2 + 2tv \cos \alpha + v^2} \quad (5)$$

$$P = \frac{2mRT_c}{V_{SE}(S - B \cos (\varnothing - a))} \quad (6)$$

In equations (1) and (2), the terms $V_{DE} + V_H$ and $V_{DC} + V_K$ represent the total dead volumes from the heater and the cooler, respectively. The volume occupied by the nitrogen inside of the regenerator was neglected [9] in comparison with the other volumes.

Fig. 2 illustrates the evolution of the pressure and the volume along the engine when the source temperature decreases. A simple analysis from a SE resolves that an increase in the source temperature represents an increment in the power output for a fixed sink temperature. The PV diagram for the three source temperatures illustrates in this figure that the work in the compression or cooler piston is maintained constant, while the work in the expansion or the heater piston is continuously reduced as the source temperature decreases.

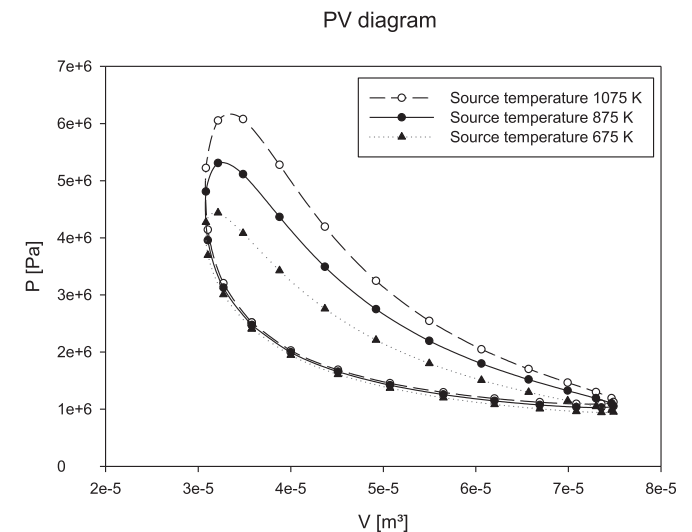
By representing the different source temperatures or the exhaust gas temperatures against the experimental and theoretical power output, a lineal relation between both of them is found. This relation is true for both the Schmidt analysis and the Whispergen unit as Fig. 3 illustrates. Expression (7) can be used to calculate the power output from a Whispergen unit using the power obtained with the Schmidt's analysis at different heat source temperatures. This expression was experimentally validated in the Whispergen unit between 640 K and 720 K.

$$P_{\text{Whispergen}} [W] = P_{\text{Schmidt}} [W] - 3.54 \cdot T_H [K] + 661.3 \quad (7)$$

Although this experimental expression could be used to calculate the power output, an expression, which reflects the influence of both the cold and hot source temperatures, is needed.

The Beale number and the West number are constants that are experimentally determined [11]. They can be utilised to approximately calculate the power output from a SE. The Beale number (8) estimates that the power output from a SE is proportional to the mean pressure, swept volume and speed.

$$B_N = \frac{P_i}{P_M \cdot V_{SE} \cdot N} \quad (8)$$

**Fig. 2.** Pressure and volume evolution when source temperature is modified.

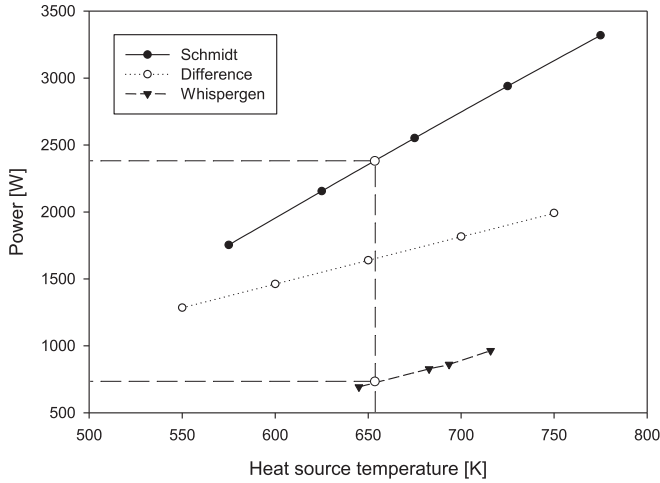


Fig. 3. Comparison between Schmidt analysis and experimental data from Whispergen unit.

When William Beale performed the tests to determine this experimental number, he used similar source temperatures. For this reason, West overcame plus experiments where the heat source and the cold source temperature were varied. A similar relation was found, even though the heat and cold sources temperatures were also in the equation (9).

$$W_S = \frac{P_i}{P_M \cdot V_{SE} \cdot N \cdot (T_E - T_C / T_E + T_C)} = B_N \left(\frac{T_E + T_C}{T_E - T_C} \right) \quad (9)$$

For this experimental number, this dependence on the temperature term is implemented in the lump model, which improves the proposed model from the previous work of the authors. The output power is obtained with the expression (10) instead of using a straight line for the stopping process similar to the preceding model. This equation uses the temperature of the two lumps with two correction constants K_1 and K_2 , which are experimentally obtained.

$$P_i = K_2 \cdot \left(\frac{T_{L1} - T_{L2}}{T_{L1} + T_{L2}} - K_1 \right) \quad (10)$$

6. Lump model

In a previous work from the authors, a transient model based on two lumps was developed within the TRNSYS simulation environment [24]. In that article, the dynamic response from the cogeneration model was principally reduced to the behaviour of two masses and one heat exchanger. The mass of these two lumps and the heat transfer coefficients were sized to obtain the temperature profiles of the exhaust gases and water outputs, respectively, when the sink temperature was varied.

New experiments were carried out on the test bench, where the air mass flow entering into the combustion chamber was varied. A reduction in the air mass flow provokes the decline of the fuel entering into the burner and consequently, the heat source temperature is reduced during these tests. Comparing these new experimental results with the results obtained in the original lump model, important discrepancies were found, and an improved model was required.

These new experiments were conducted by looking for a mechanism to vary the power output from the engine. This model of the cogeneration device allows us to indirectly control this

power output from the engine by varying the hot source temperature in the combustion chamber.

As explained previously, the power output from this Stirling engine is controlled via the airflow entering into the burner because the fuel pump introduces diesel into the combustion chamber for a control system. This control measures the oxygen content in the exhaust gases, and it keeps a prefix fuel-air ratio. Depending on this airflow, different hot source temperatures are reached.

By varying the excitation power into the brushless alternator [29], the revolutions from the SE are practically kept constant during the running mode. This process provokes a constant thermal resistance and consequently, the heat flow, which is moved by the nitrogen inside the pistons between the hot and cold source, depends principally on the difference in the temperatures between the nitrogen in the cold side and hot side and the temperatures from the sink source and heat source.

In the expansion space (hot side), the heat flow can be obtained with the expression (11):

$$\dot{q}_{in} = \frac{1}{R_H} \Delta T_H = \frac{1}{R_H} (T_H - T_U) \quad (11)$$

where R_H represents the thermal resistance from the hot side. Although this resistance depends on different factors (such as frequency, properties from the nitrogen, running temperatures or the heating method), its value can be considered constant [12] to simplify the SE analysis. In Ref. [9], this heat flow is calculated using the expression (12), where h_H represents the convection coefficient.

$$\dot{Q}_{in} = h_H A_H (T_H - T_U) \quad (12)$$

Following the same argument for the cold side, the heat flow that the nitrogen releases into the sink source can be calculated with the expression (13) or similarly (14). The principal heat flows are depicted in Fig. 4.

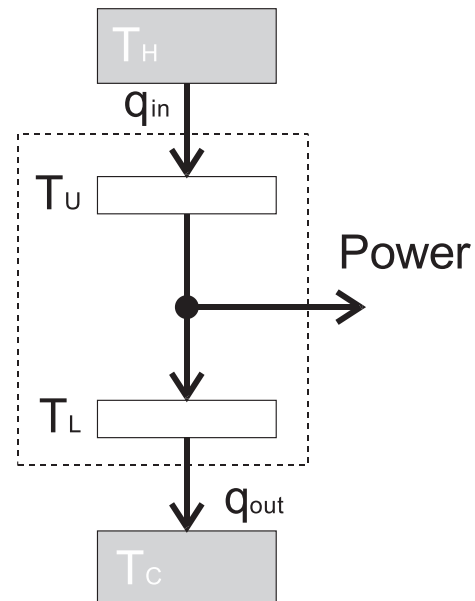


Fig. 4. Temperatures and heat flows into the SE unit.

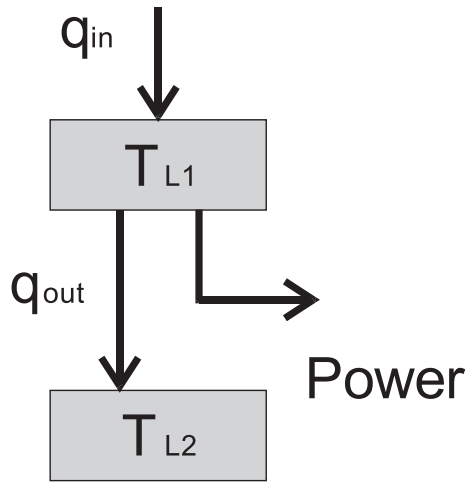


Fig. 5. Reduced system.

$$\dot{q}_{out} = \frac{1}{R_C} \Delta T_C = \frac{1}{R_C} (T_L - T_C) \quad (13)$$

$$\dot{Q}_{out} = h_C A_C (T_L - T_C) \quad (14)$$

Consequently, the values of $\dot{q}_{in}$ and $\dot{q}_{out}$ will depend principally on the difference in the temperatures between the hot side temperature and the nitrogen temperature into the expansion space and on the difference in the temperatures between the sink temperature and the temperature from the nitrogen within the compression space as shown in Fig. 4. Different tests were performed in the cogeneration unit, and they showed that the engine operates at a similar speed during the starting and the steady state periods. However, during the stopping period, the revolutions from the engine suddenly drop.

Therefore, the performance of the global system can be reduced to the evolution around the time of the two lumps as represented in Fig. 5. These lumps represent a simplified version of the SE, and they will replicate the behaviour of the real system by means of a method, which could be applied to other machines.

The reduced equation (15) was used to represent the heat flow, which circulates between the lumps, and it uses an equivalent heat transfer coefficient.

$$\dot{Q}_{out} = \dot{Q}_{lumps} = (UA)_{lumps} (T_{L1} - T_{L2}) \quad (15)$$

Therefore, because the SE Whispergen unit runs with a similar rotating speed during the starting process and during the steady state period, it used the same heat transfer coefficient for these periods. By contrast, during the stopping process, it used a different coefficient of transference to take into account the sudden drop of the working speed and consequently the variation of the heat flow between the heat and cold sources.

Although the speed reduction decreases the times per second that the nitrogen release or the capture heat from the sink or the heat source, respectively, the period of time to exchange this heat increases and provokes a bigger coefficient of transference. Therefore, the engine stops quickly when the fuel entering into the combustion chamber is cut off.

The expression (16) is introduced in the TRNSYS simulation environment, and it controls the heat flow between the two lumps' model, which represent the heat exchange between the expansion chamber and the compression chamber from the Whispergen unit.

$$\dot{Q}_{lumps} \begin{cases} (T_{L1} - T_{L2}) * (UA)_{lumps_1} & \begin{cases} m_{fuel} < > 0 \\ T_{L1} > 413 \end{cases} \\ (T_{L1} - T_{L2}) * (UA)_{lumps_2} & \begin{cases} m_{fuel} = 0 \\ T_{L1} > 543 \end{cases} \\ 0 & \text{in other case} \end{cases} \quad (16)$$

This expression also contains information about the performance and control logic of the SE. The heat flow between the two lumps is allowed when different conditions are fulfilled. However, when the exhaust gas temperatures are greater than 140 °C, the engine starts to turn, and the heat exchange between the lumps is allowed. When there is no fuel going into the combustion chamber but the exhaust gases temperature is still over 270 °C, the heat exchange between both lumps is also allowed. These conditions correspond to the logical operation of the SE and are fulfilled in all of the undertaken experiments in the cogeneration unit.

Fig. 6 illustrates the cogeneration unit that is simulated in the TRNSYS simulation environment. L1 and L2 represent the lump units that simulate the behaviour of the SE, and ST_HE represents the heat exchanger placed in the exhaust gases.

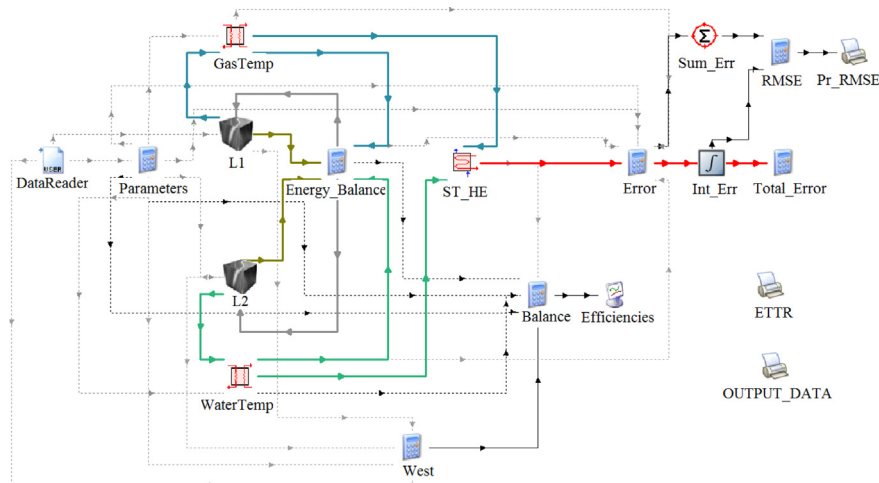


Fig. 6. Schematic of the propose model in the TRNSYS simulation environment.

7. Heat exchanger arrangement into the cogeneration plant

The original Whispergen unit uses an external heat exchanger to recover the heat flow within the exhaust gases. When the SE reaches the steady-state regime, the exhaust gases reach temperatures of approximately 400 °C. This heat flow represents an important quantity of energy compared with the energy introduced by means of the fuel inlet. For this reason, the Whispergen unit recovers this heat flow by means of a shell and tube heat exchanger using the cooling water (sink source) going out of the block and consequently, the exhaust gas temperatures decrease by approximately 80 °C.

Fig. 7 illustrates the original distribution of the heat exchanger within the cogeneration unit. A SE is primarily formed by three heat exchangers (heater, cooler and regenerator) that are carefully designed to fit the compromise between different factors, such as pressure drop or heat exchange. The heat exchanger that corresponds to the regenerator was not represented in Fig. 7 because this regeneration is considered perfect, and it corresponds with the internal performance of the SE that is not taken into account in this analysis. The equations (17)–(22) represent the SE behaviour and the principal exchanger energy inside the cogeneration system.

$$Q_{HE1} = \dot{m}_{gas} \cdot c_{p_{gas}} \cdot (T_{L1} - T_{gas}) = \dot{m}_{water} \cdot c_{p_{water}} \cdot (T_{out} - T_{L2}) \quad (17)$$

$$\dot{m}_{gas} = \dot{m}_{air} + \dot{m}_{fuel} \quad (18)$$

$$MC_{L1} \frac{dT_{L1}}{dt} = \dot{m}_{fuel} \cdot LHV - \dot{m}_g c_{p_{gas}} (T_{og} - T_{ig}) - (UA)_{lumps} (T_{L1} - T_{L2}) - Power \quad (19)$$

$$MC_{L2} \frac{dT_{L2}}{dt} = (UA)_{lumps} (T_{L1} - T_{L2}) - \dot{m}_{water} c_{p_{water}} (T_{L2} - T_{ret}) \quad (20)$$

$$Q_{comb} = \dot{m}_{fuel} \cdot LHV \quad (21)$$

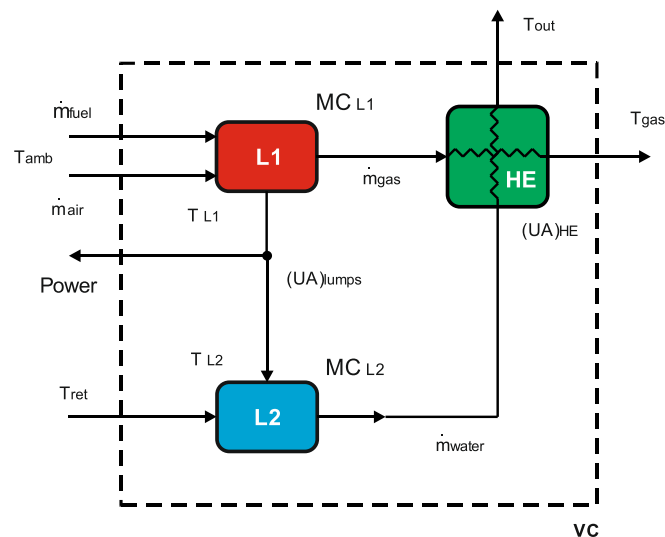


Fig. 7. Original heat exchanger distribution.

$$Power = W_{net} \cdot n \quad (22)$$

The new configuration represented in Fig. 8 was simulated for the purpose of investigating the ETTR variation. This simulation was carried out in the TRNSYS simulation environment, and the obtained results were compared with the original data from the cogeneration plant that is represented in Fig. 7. By preheating the mass flow air that it is entering into the combustion burner, an important reduction in the fuel consumption can be obtained for the same power output. This reduction in fuel consumption is especially interesting when the system is tracking the electrical load.

Equation (23) represents the equilibrium in the newly proposed heat exchanger. This final exchanger preheats the ambient air up to temperatures near the exhaust gases from the combustion chamber, and the UA from the heat exchanger was selected to maintain the same gas outlet temperature.

$$Q_{HE2} = \dot{m}_{gas} \cdot c_{p_{gas}} \cdot (T_{L1} - T_{gas}) = \dot{m}_{air} \cdot c_{p_{air}} \cdot (T_{in} - T_{amb}) \quad (23)$$

8. Calibration methodology

The capacitances from the block and from the burner, the coefficient of transference between the lumps, the coefficient of transference in the final heat exchanger and the constants k_1 and k_2 from the power output equation were introduced as input variables in the model. They were obtained through an iterative process using TRNSYS and GenOpt. This optimisation program tries to minimise the objective cost function using a predefined algorithm. The selected cost function was the RMSE of the principal values from the SE: the power output, the exhaust gas temperatures from the burner and the temperature of the block. Because the input variables are continuous and the cost function is differentiable, the Hooke–Jeeves algorithm can be used to search for the best result. During the iterative process, all the input values were varied between acceptable values.

This calibration process was performed using data from the tests with ninety percent of the air mass flow, and the model was

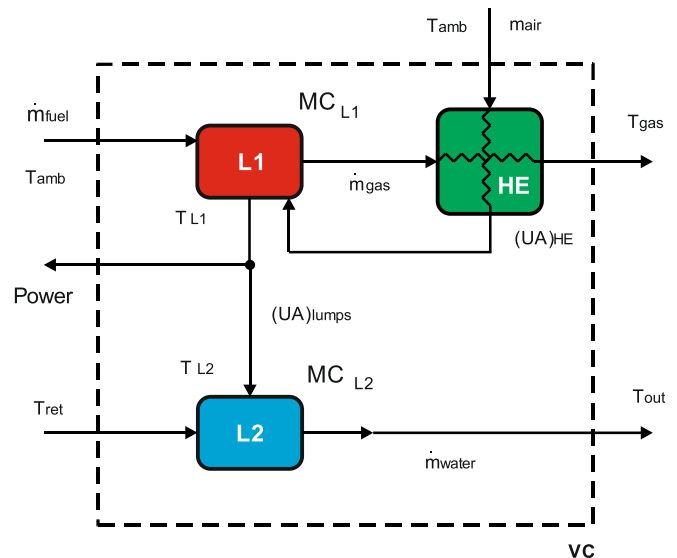


Fig. 8. New proposal for the heat exchanger distribution inside the system.

validated with the experimental data from the tests with seventy, eighty and one hundred percent of the air mass flow.

9. Results and discussion

9.1. Experimental results

For all of these dependencies and with the aim of validating the newly proposed model, different experiments were performed to collect the performance of the Stirling engine at different heat source temperatures. Because the sink source temperature depends principally on the heating system, all experiments were performed with a constant sink source temperature of 60 °C.

The collected data from these experiments are represented in Figs. 9–11. In Fig. 9, it can be observed that the heat source temperatures in the steady state period for the different working points are in the range between 350 and 450 [°C].

During the first fifteen minutes, the Whispergen engine presents the same behaviour and is independent of the set point for the airflow. During the starting process, the engine checks different sensors and parameters to ensure a good starting process, and it follows the same starting ramp until minute 14 is reached. Furthermore, once the exhaust gases from the burner achieve a temperature of 150 °C, the engine is forced to turn.

For the power output, these differences are greater as shown in Fig. 10. At maximum power, the airflow is set at 100%, and the power output reaches values of approximately 1 kW. When the airflow is set to 70%, the power output in the steady state period has a value near 700 W.

However, when the cogeneration system is forced to work with a 60% quantity of air, the power output is not sufficient to supply the consumption of energy from the test bench, and the engine has to stop to avoid the discharge of the batteries. For this reason, the collected data from the test with 60% air is represented, but it is not considered within the simulations.

The voltage tracking from the system keeps the frequency from the alternator constant and consequently the speed from the engine. The brushless alternator from the Whispergen unit produces a three-phase alternating energy. The frequency of this current was measured by an oscilloscope during the experiments, and it was found that the values were in the range 150–170 Hz in the starting and steady-state periods. These values correspond to a speed between 1500 and 1700 rpm for expression (24), where P represents the number of poles per phase.

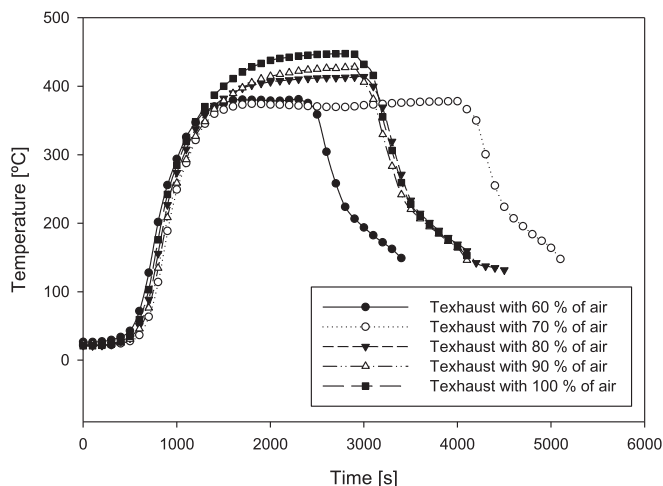


Fig. 9. Exhaust gases temperatures running at different mass flow inlet.

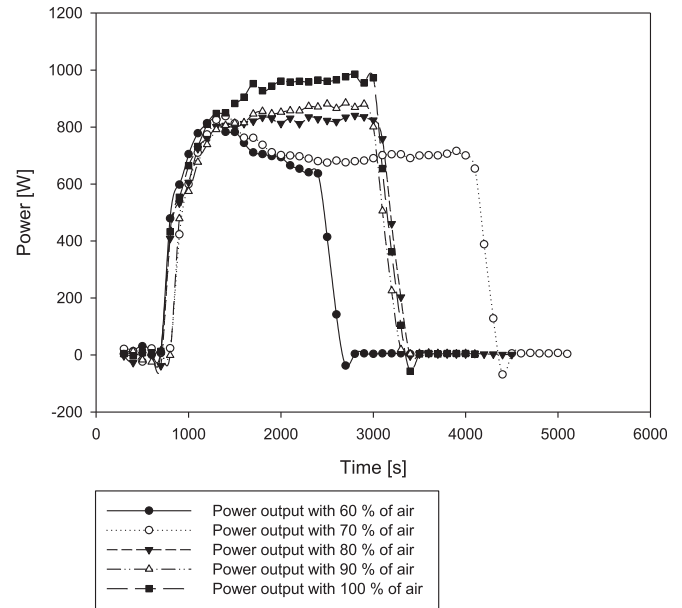


Fig. 10. Power output working at different air mass flow.

$$N [\text{rpm}] = \frac{120 \cdot n [\text{Hz}]}{P} \quad (24)$$

The three-phase energy is converted into direct current with a rectifier, which assures the correct charging process for the battery bank at 24 V DC. Because the voltage from the system is maintained constant, the intensity going out of the SE follows the same variation as the power output as shown in Fig. 11.

Despite the differences for the source temperatures and the power output, the power efficiency from the engine approaches 10%, and there is no clear disparity between the variety of running points. Due to the thermal inertia from the system, this efficiency is greater in the test of 60% and 70% air during the starting period, but

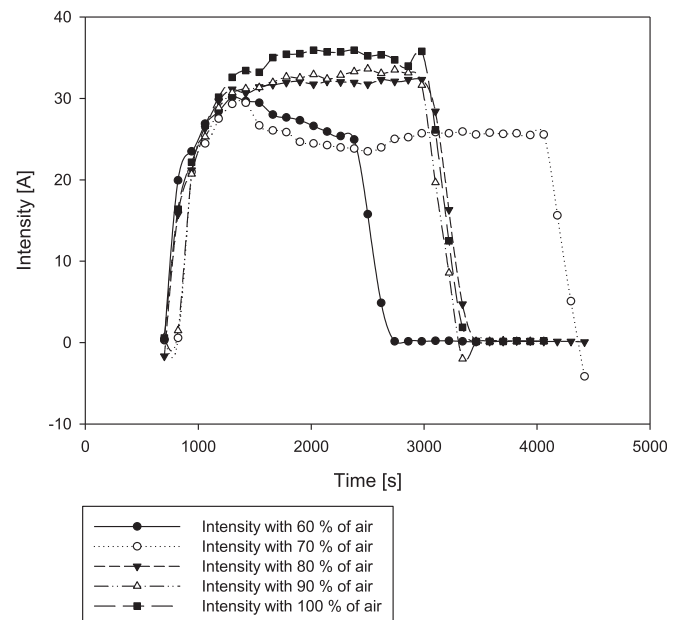


Fig. 11. Intensity from the alternator working at different source temperatures.

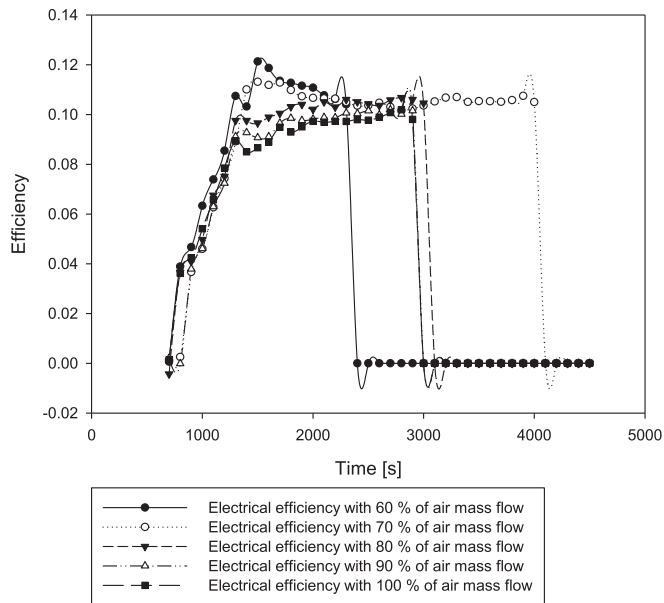


Fig. 12. Electrical efficiencies at different air mass flows.

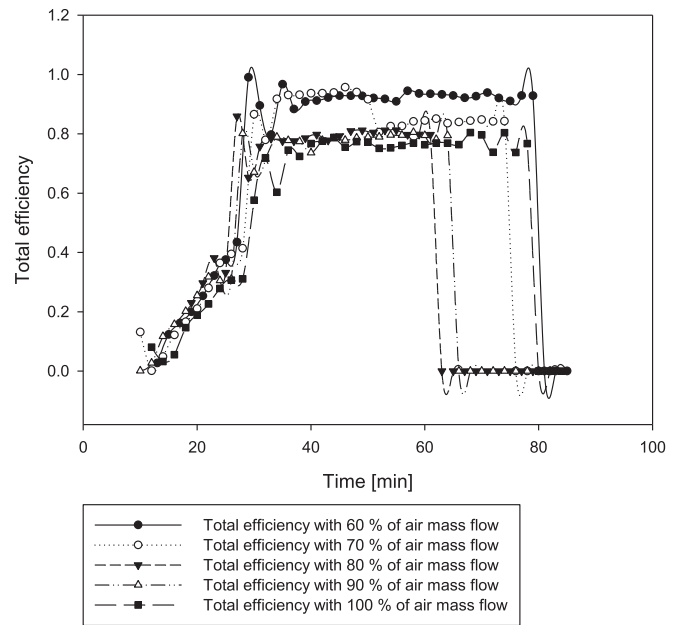


Fig. 14. Total efficiencies at different air mass flows.

once the steady state is reached, these differences are reduced as Fig. 12 illustrates. As the air mass flow increases, the power efficiencies are only slightly larger, but these differences are not sufficient to extract a valid conclusion, and they cannot be used to vary the ETTR ratio.

In Figs. 12 and 13, the obtained electrical and thermal efficiencies measured at different air mass flow inputs are illustrated. These experimental data show that the engine tends to work at a fixed working point that is independent of the source temperature. As the air mass flow decreases, the thermal efficiency slightly increases, but the power output drops below the critical value of 600 W, where the SE is forced to stop.

Figs. 13 and 14 show that after 30 min of running time, the cogeneration system starts to produce a considerable quantity of

heat with a good efficiency. This transient performance shows the importance of developing transient models that are easy to integrate into simulation tools because the starting time is an important period compared with the total running time especially if the cogeneration system is not oversized, and it is turned on several times per day.

For the stopping time, Fig. 15 and Table 2 represent the total energy produced during this downtime. This energy is also important to consider, especially during the first ten minutes, when the engine is still turning.

The ETTR ratio from the simulated Whispergen unit represents the relation between the total power output and the total thermal energy obtained through the SE cogeneration device, and it can be calculated with expression (25).

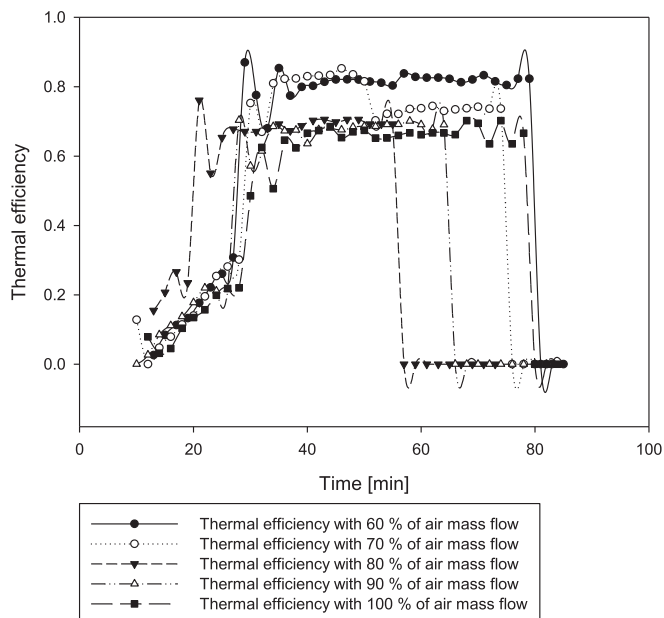


Fig. 13. Thermal efficiencies at different air mass flows.

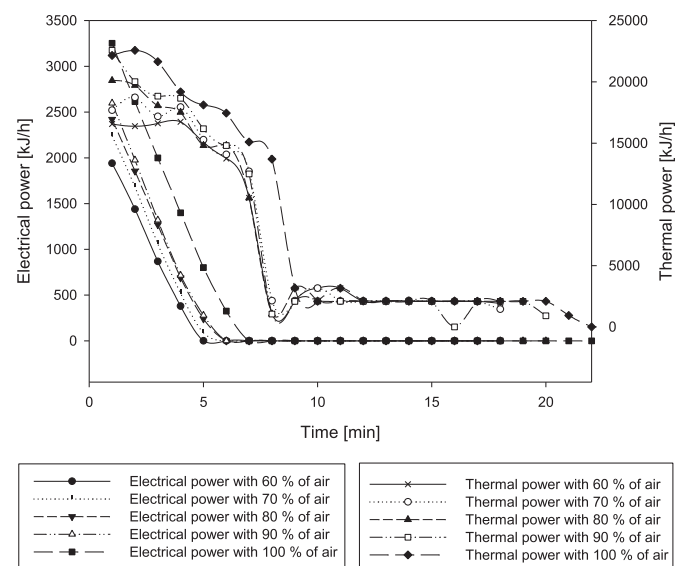


Fig. 15. Electrical and thermal power for the different test during the stopping time.

Table 2
Thermal and electrical power during the stopping time.

	60% Air	70% Air	80% Air	90% Air	100% Air
Power [kWh]	0.041	0.052	0.064	0.068	0.121
Q _{th} [kWh]	3.28	3.37	3.17	3.54	4.75

$$ETTR = \frac{P_i}{P_{th}} \quad (25)$$

The total thermal energy P_{th} is measured by means of the circulating water circuit using the specific heat from the water, the mass flow and the difference in the temperatures between the inlet and the outlet as expressed in equation (26).

$$P_{th} = \dot{m}_{water} \cdot c_{p_{water}} \cdot (T_{out} - T_{ret}) \quad (26)$$

Using both expressions, the ETTR ratio is calculated for all the different experiments, and it is illustrated in Fig. 16. This ratio is approximately 0.15 using 80–100% of the maximum mass flow air. As the mass flow is reduced, this ratio slightly decreases, even though no important variations are obtained.

9.2. Calibration results

Table 3 represents the obtained results in the optimisation process, where different inputs were varied for the calculated cost function considered in the calibration process.

Fig. 17 represents the experimental and simulated data from the test with ninety percent of the air. In this figure, the output power and the exhaust gas temperatures at the burner output are depicted. In Fig. 18, the experimental and simulated data from the test with 70 percent of the air are also represented. This simulation was performed with the values of the capacitances and the coefficients of transference that were obtained with the experimental data from the test with 90 percent of the air, and the obtained results show that the model adopted is valid with different air mass flows.

Similar agreements are found within the tests of 80 and 100% of the air. Table 4 summarises the obtained RMSE from the different tests. It can be checked that the water outlet temperature, the gas outlet temperature and the power output, which represent the most important outputs, are predicted with an error less than 5%.

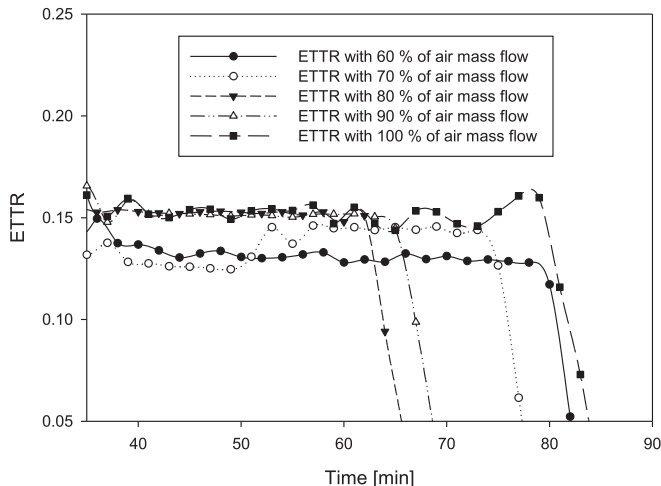


Fig. 16. ETTR along the different test during steady state period.

Table 3
Optimisation results.

Calibration results		
Capacitance_block	54.24	[kJ/K]
Capacitance_burner	12.86	[kJ/K]
Ualumps_starting-steady-state	10.59	[W/K]
Ualumps_stopping	15.65	[W/K]
UA_final	20.04	[W/K]
k1	0.19	[–]
K2	20,838.69	[–]

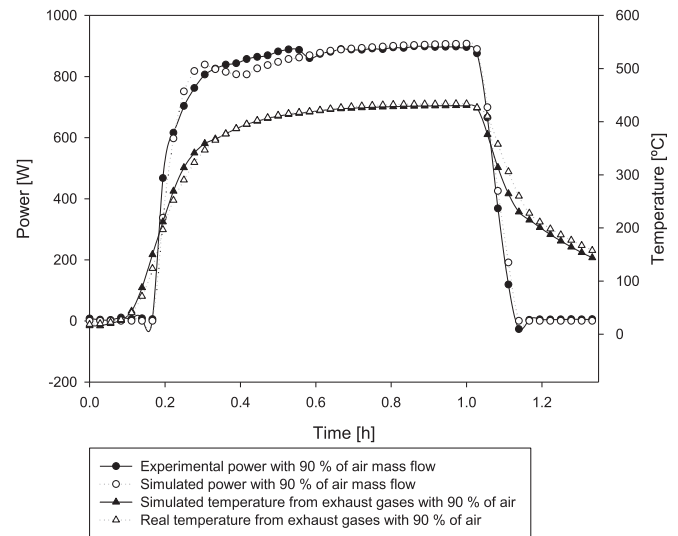


Fig. 17. Experimental and simulated data from the power output and exhaust gases temperature at 90% air.

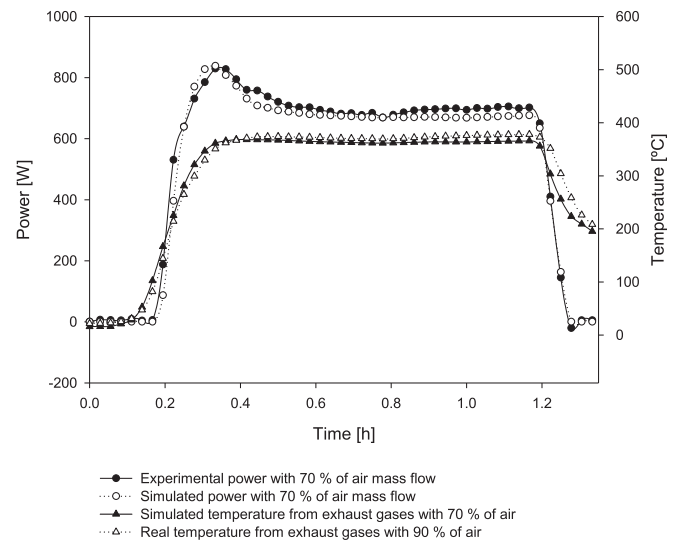


Fig. 18. Experimental and simulated data from the power output and exhaust gases temperature at 70% air.

Table 4
RMSE of the principal outputs from the different tests.

% Air	Gases	Water	Power
70	3.78%	1.73%	0.28%
80	3.95%	2.32%	0.37%
90	3.63%	1.64%	0.27%
100	3.88%	1.78%	0.30%

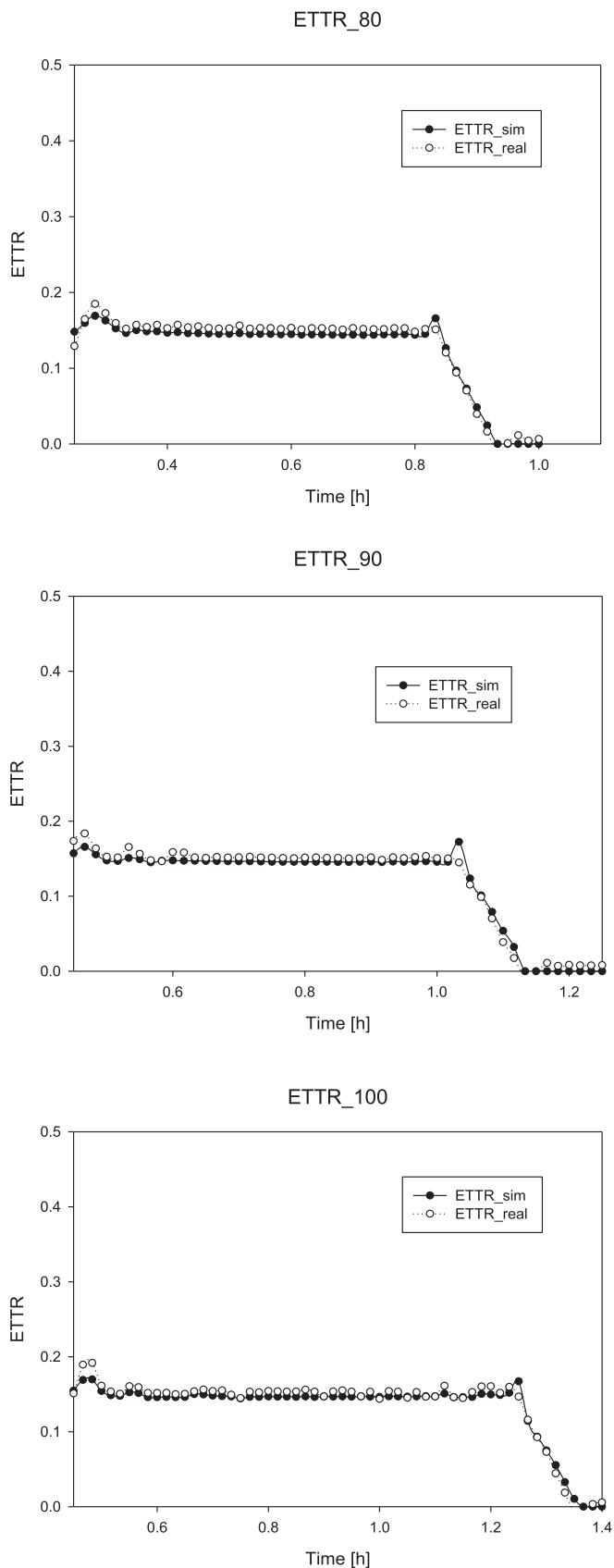


Fig. 19. Experimental and simulated ETTR from test with 80%, 90% and 100% of air.

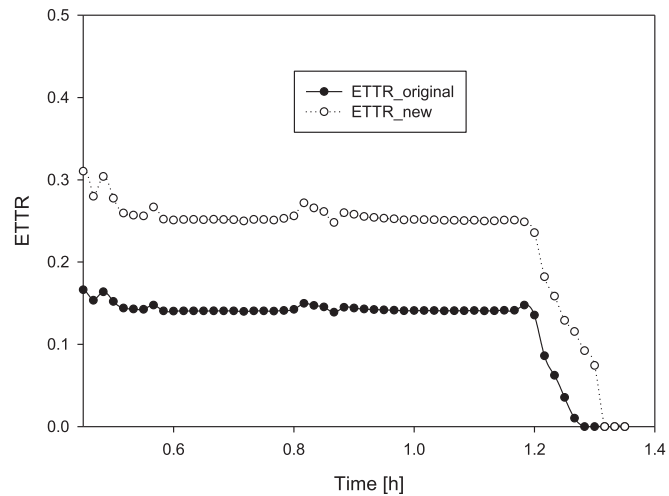


Fig. 20. ETTR for the old and new configuration with 70% of air mass flow.

These results demonstrate that the values of the capacitances and the coefficients of transference that are obtained through the optimisation process are valid to simulate the behaviour of all possible transient behaviour from our SE.

Fig. 19 represents the experimental and simulated ETTR ratios for the experiments with 80%, 90% and 100% of the air mass flow. These figures show a good agreement between the real and simulated ETTR data during the steady-state period. They are all approximately 0.15 as explained previously.

9.3. Results of the modified SE cogeneration unit

Fig. 20 represents the ETTR ratio from the original cogeneration plant, and it is compared with the ETTR ratio obtained in the simulation of the new design. This comparison was made for the test with 70% of the air mass flow.

Similar results were obtained for the rest of the tests for the different air mass flows. These results are represented in Table 5. In the original distribution, the ETTR ratio is near 0.15 for all the experiments. However, with the new distribution, an ETTR ratio near 0.26 is obtained for all the simulated cases. The simulations were performed with the same mass fuel inputs obtained from all the experiments and that correspond to each air mass flow rate.

Figs. 21–24 represent the obtained results for the different analysed cases. In these figures, the simulated power of the original distribution is compared with the simulated power from the new arrangement. Moreover, the principal outlet temperatures are also compared.

The values corresponding to the steady-state values for the original distributions and the new distributions are summarised in Table 6.

In general, due to the increase of temperature in the exhaust gases from the combustion chamber, the temperature in the expansion space also increases and corresponds with an increment for the difference in the temperatures between the cold source and

Table 5
ETTR from the original and the new heat exchanger distribution.

	% Air	ETTR_original	ETTR_new
Original distribution	70	0.143	0.257
	80	0.147	0.265
	90	0.148	0.268
	100	0.148	0.264

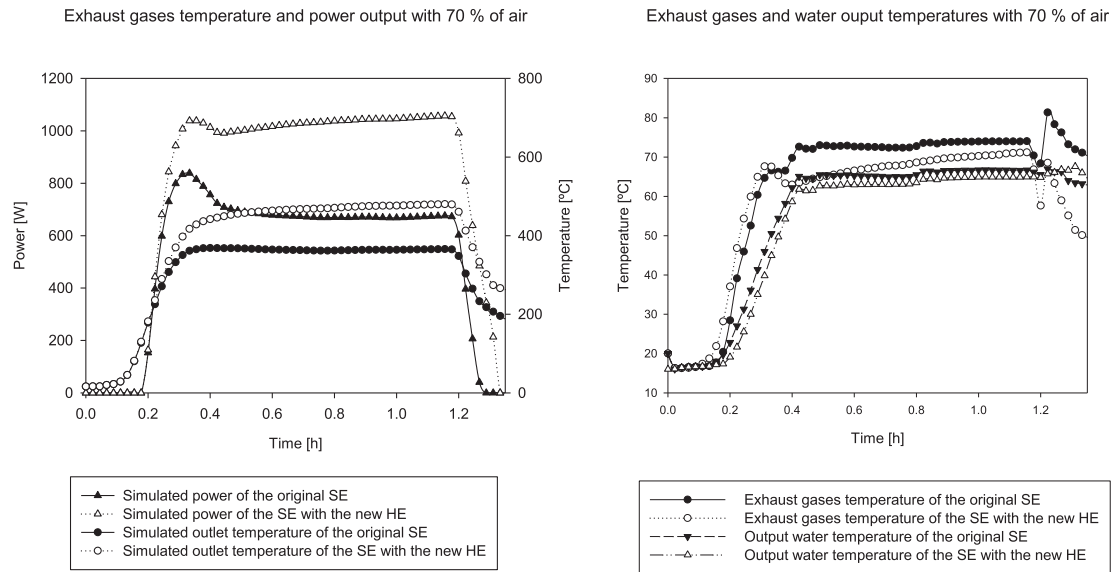


Fig. 21. Power output and principal outlet temperatures with 70% of air mass flow rate.

the heat source, which directly implies an augmentation on the theoretical efficiency corresponding to Carnot's theorem.

The security system from the Whispergen unit does not allow an increase in the exhaust gases temperature of over 480 °C, and the engine is forced to stop if this happens. This protocol assures the preservation of the sealing and assures that the working gas is kept inside the cylinders.

Table 6 shows that with the new distribution, a power of approximately 1 kW can be reached with 70% of the air mass flow and supposes a 30% reduction in the fuel consumption to produce the same power output as 100% of the air mass flow in the original SE distribution.

These results reveal that an important fuel saving could be obtained with the new distribution in the cogeneration plant. This new arrangement becomes especially interesting if the cogeneration unit is for an electrical load or can toggle between the heating and charging mode similar to the Whispergen unit.

It is important to remember that this is a preliminary analysis, where the different pressure losses or the new heat exchanger parameters corresponding to the nitrogen working at higher temperatures were not taken into account.

Other heat exchanger distributions similar to Fig. 25 can obtain an intermediate ETTR ratio if desired. Furthermore, this ETTR ratio could easily be made variable with an adequate system of linkage, which could track the different loads that increase the dynamic response from the model.

10. Conclusions

In this article, the performance of a cogeneration unit that is principally composed of a SE, an alternator and a heat exchanger is studied. The obtained experimental results that work at different mass fuel inputs are presented and analysed. These tests correspond to a variation in the source temperature.

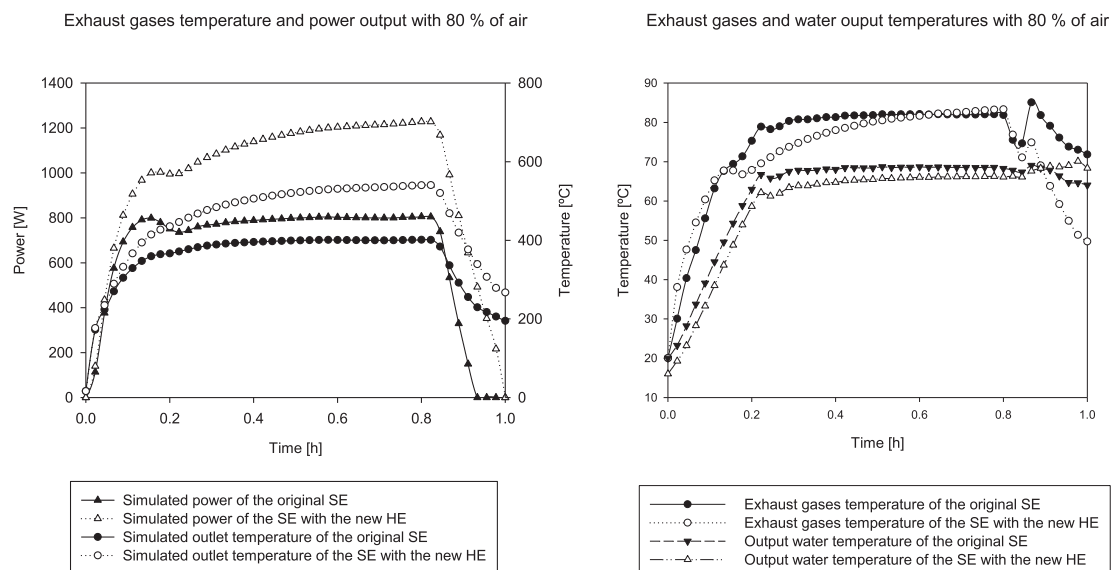


Fig. 22. Power output and principal outlet temperatures with 80% of air mass flow rate.

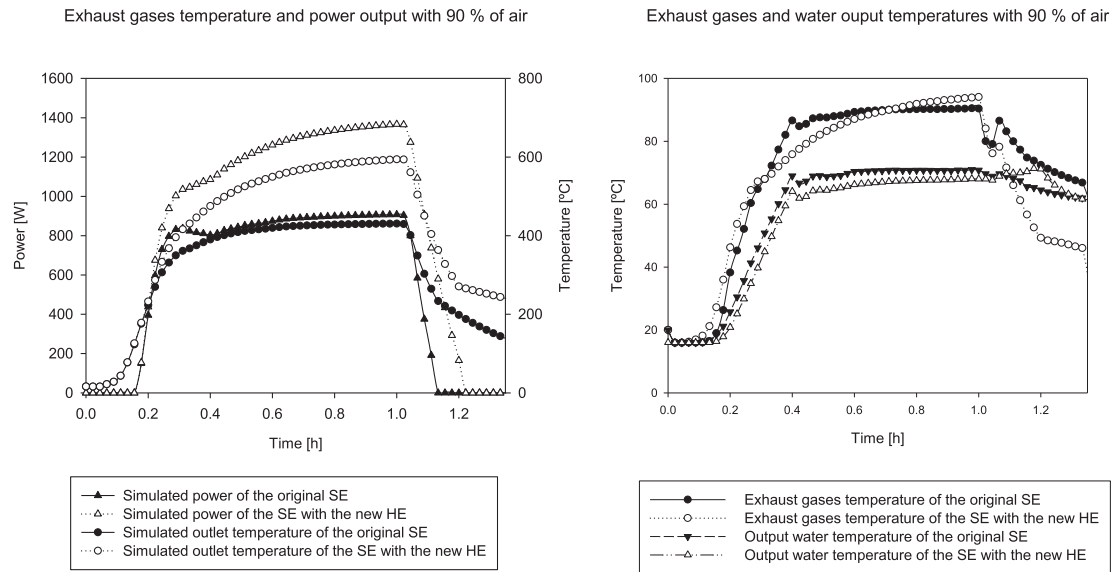


Fig. 23. Power output and principal outlet temperatures with 90% of air mass flow rate.

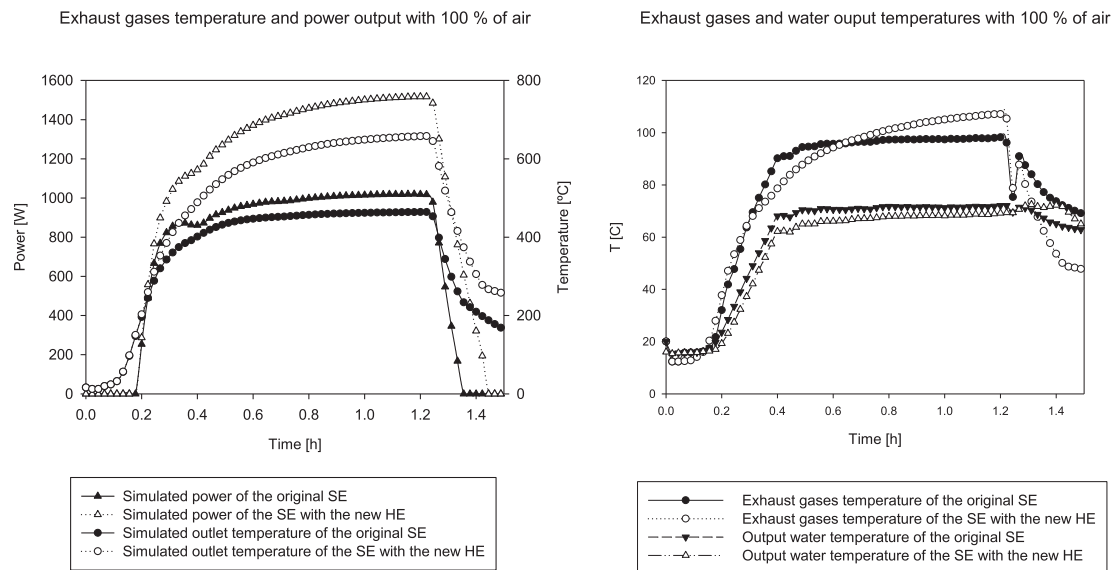


Fig. 24. Power output and principal outlet temperatures with 100% of air mass flow rate.

A new transient model of a Stirling engine based on two lumps and one heat exchanger is presented. This model is adapted to the different transient states that correspond to different air mass flow inputs.

Table 6
Steady state values for the most important variables.

	% Air	$\dot{m}_{\text{fuel}}$ [kg/h]	T_{out} [°C]	T_{exh} [°C]	T_{gas} [°C]	Power [W]
Original distribution	70	0.56	64.73	72.49	364.60	691.33
	80	0.64	68.35	81.45	397.96	795.33
	90	0.73	69.97	89.03	420.16	876.95
	100	0.81	71.26	96.81	455.04	993.43
New distribution	70	0.56	62.30	67.71	464.27	1030.11
	80	0.64	65.48	79.97	520.46	1177.42
	90	0.73	66.21	87.89	555.34	1274.58
	100	0.81	67.48	100.38	623.20	1443.44

Furthermore, the starting and stopping periods are better simulated than the new relations founded in the model. The heat flow between the different lumps was also accommodated to the performance of the model, and the different heat transfer coefficients were calculated for the engine speed.

The data obtained from the experiments were used to calibrate the model and had a RMSE value below 0.4% for the generated power.

The new model has been used to investigate the behaviour of the SE when the thermal energy from the exhaust gases is used to preheat the air in the combustion chamber. This mechanism allows the increase of the electrical efficiency and an important fuel saving when the system tracks the power consumption.

The air mass flow rate entering into the burner was also varied, and the real power output from the engine was analysed together with the electrical and thermal efficiencies.

